B.Sc(H) part III

Paper :- VII

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Group A

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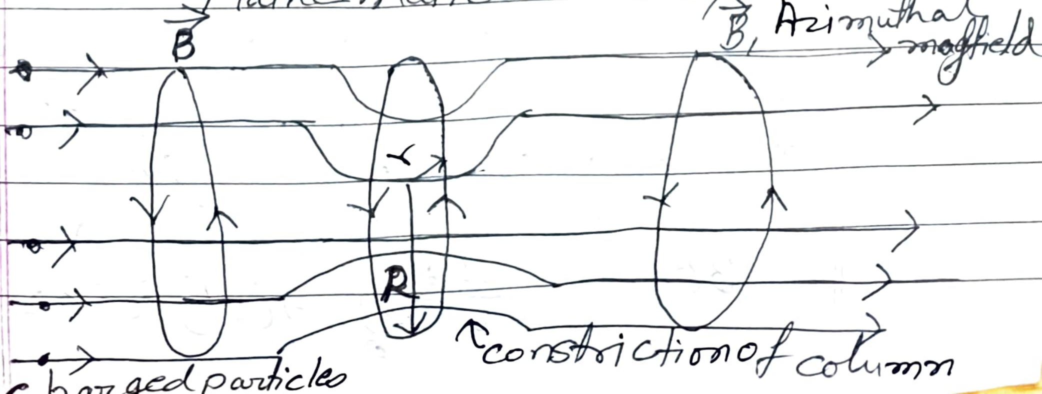
"PINCH EFFECT"

"pinch effect is the confinement of plasma by self magnetic field"

A large current through an ionised gas creates a strong magnetic field. If the current flows in a cylindrical column the lines of force from concentric circles with the cylinder. The lateral repulsion between lines exerts a pressure radially inwards. As the charge carriers are free to move in space, the discharge is consequently compressed inwards. This squeezing or pinching of the column of streaming particles or plasma is known as "PINCH EFFECT"

Mathematical Analysis :-



Currentdensity due to the streaming particles

$$\vec{J} = \rho v$$

Where ρ = volume charged-ensity

v = drift velocity

Let $\vec{B}$ = magnetic field produced by the current then magnetic force on unit volum is

$$\vec{F} = \vec{J} \times \vec{B} \quad \text{——} \; ①$$

→ $\vec{F}$ is direacted ⊥r $\vec{J}\times\vec{B}$ towards the centre of the column, so the ~~strongma~~ streaming particles experinced force in radially inward direction & hence the particles get squeezed or pinched.

From Ampere's circuital law, The magnetic fied at a distance r from the axis of the discharge is given as

$$B(r) = \frac{\mu_0}{r}\int_0^r J(r')\,r'\,dr' \quad \text{——} \; ②$$

Now,

$$\frac{\partial B(r)}{\partial r} = -\frac{\mu_0}{r^2}\int_0^r J(r')r'dr' + \frac{\mu_0}{\cancel{r}} J(r)\cancel{r}$$

$$= -\frac{\mu_0}{r^2}\int_0^r J(r')r'dr' + \mu_0 J(r)$$

Using equⁿ ②

$$\frac{\partial B}{\partial r} = -\frac{1}{r}B(r) + \mu_0 J(r)$$

$$\Rightarrow \mu_0 J(r) = \frac{\partial B}{\partial r} + \frac{B(r)}{r}$$

$$\Rightarrow J(r) = \frac{1}{\mu_0}\frac{\partial B}{\partial r} + \frac{B(r)}{\mu_0 r} \quad ③$$

The magnetic force per unit volume is

$$\vec{F} = \vec{J} \times \vec{B}$$

$$= -J(r)B(r)\hat{r}$$

$$\therefore |\vec{F}| = F = -J(r)B(r) \quad ④$$

using equⁿ ② in equⁿ ④

$$F = -\frac{1}{\mu_0}B\frac{\partial B}{\partial r} - \frac{1}{\mu_0 r}B^2 \quad ⑤$$

In terms of equivalant pressure p', we can write it as

$$F = -\nabla p' = -\frac{\partial p'}{\partial r} \quad ⑥$$

Equating eqn ⑤ & ⑥ we have

$$\frac{\partial p'}{\partial r} = \frac{1}{\mu_0} B \frac{\partial B}{\partial r} + \frac{B^2}{\mu_0 r}$$

Integrating this equestion with respect to r.

$$\int dp = \int_0^r \frac{B^2}{\mu_0 r} dr + \int_0^B \frac{1}{\mu_0} B dB$$

where $B = 0$ at $r = r_0$

$$\Rightarrow p' = \frac{B^2}{2\mu_0} + \frac{1}{\mu_0} \int_0^r \frac{B^2}{r} dr \quad ⑦$$

at $r = R$

plasma is a conducting fluied → high conductivity → not appreciable penetration of magnetic lines of force in it.

$$\Rightarrow \int_0^r \frac{B^2}{r} dr = 0 \Rightarrow$$ pressure inside the column is zero.

So, the equivalent pressure at the boundry of the column, $r = R$.

$$\boxed{p' = \frac{B^2(R)}{2\mu_0} = p_m} = \text{magnetic pressure}$$

As the pressure inside is zero, so column is pinched.

* * *

→ Squeezing stops when the plasma fluid pressure ~~bal~~ balanced the magnetic pressure.

Pinch Current :—

plasma fluid is treated as a perfect gas so, equ^n of state is given as

$$p = NKT$$

Where N = no. of particles per unit volum.

K = Boltzman constant.

T = absolute temperature.

So, at $r = R$ we have

$$\frac{B^2(R)}{2\mu_0} = NKT$$

Now $B(R) = \frac{\mu_0 I}{2\pi R}$

$$\therefore \frac{1}{2\mu_0} \times \frac{\mu_0^2 I^2}{4\pi^2 R} = NKT$$

$$\Rightarrow \frac{1}{2} \frac{\mu_0 I^2}{4\pi^2 R^2} = NKT$$

$$\therefore I^2 = 2\left(\frac{\mu_0}{4\pi}\right)^{-1} (\pi R^2) \times NKT$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)^{-1} A_0 N_0 KT \qquad \text{———} ⑨$$

Here $\underline{A_0 N_0} = \underline{\pi R^2 N}$

A_0 = Area of Cross section of the column

Conservation of particles

i.e A_0 = Initial crossection of the discharge

N_0 = initial partical density.

Let $T = 10^8 K$, $A_0 = 0.04 m^2$ and $N_0 = 10^2$ particles/m^3

$$\therefore I^2 = 2(10^{-7})^{-1} \times 0.04 \times 10^{21} \times 1.38 \times 10^{-23} \times 10^8$$

$$= 1.1 \times 10^{12} A^2$$

i.e $\boxed{I = 10^6 A}$

This is the discharge current needed for pinch effect

——x—— The end ——x——